Khovanov Homology of the $P(\pm l, m, n)$ pretzel links

KH of $P(\pm l, m, n)$ pretzels

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Definition: n-Manifold M

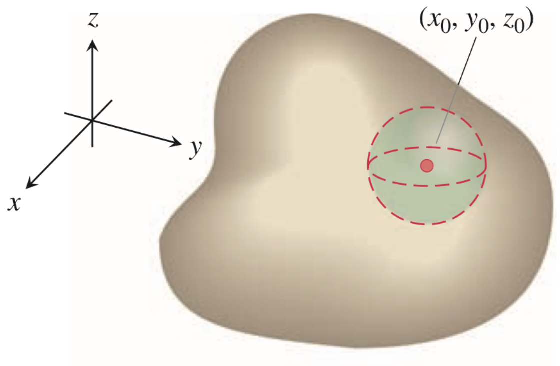

Figure: Interior point: neighborhoods homeomorphic to $\mathbb{R}^n$.

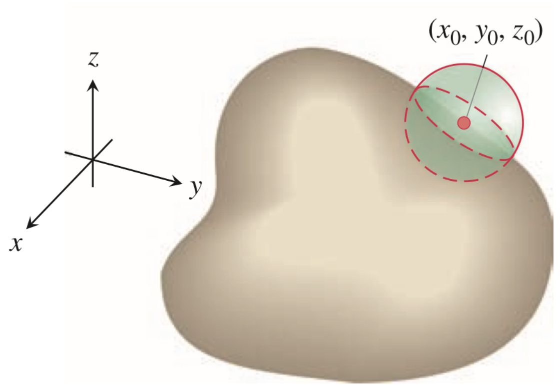

Figure: Boundary point: neighborhoods homeomorphic to $\mathbb{R}^n_+$.

- separable, metric space

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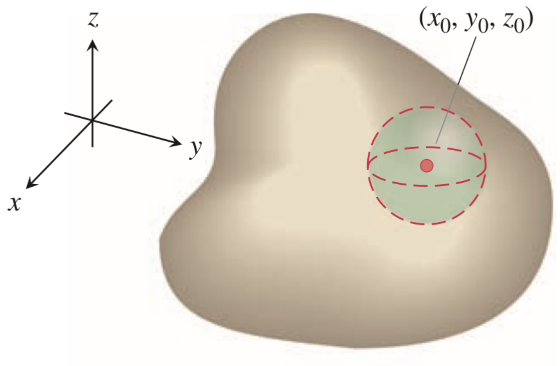

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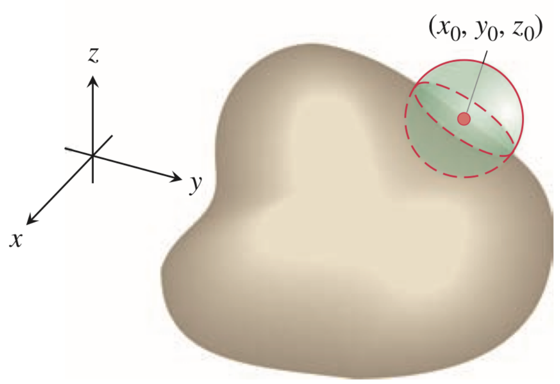

Figure: Boundary point: neighborhoods homeomorphic to $\mathbb{R}^n_+$.

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- neighborhoods homeomorphic to $\mathbb{R}^n$ o $\mathbb{R}^n_+$

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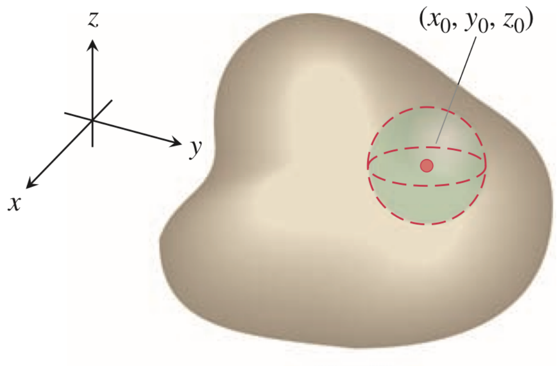

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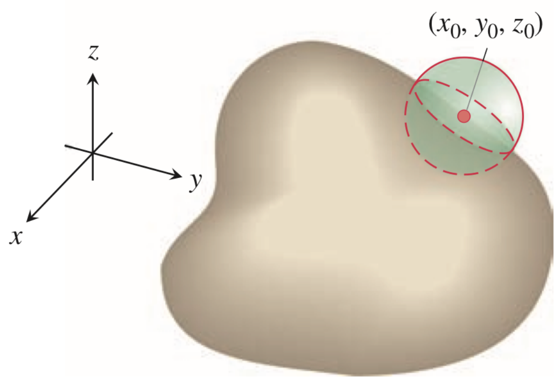

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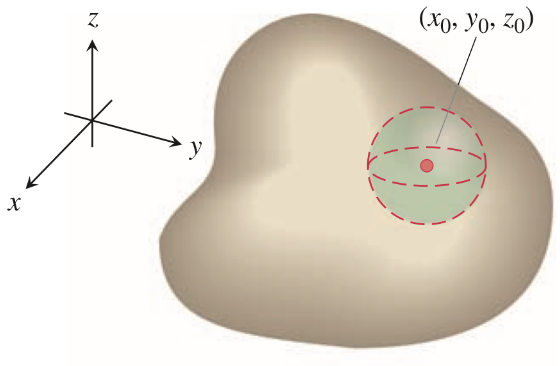

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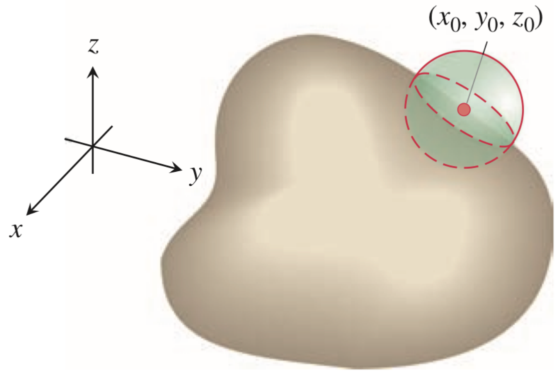

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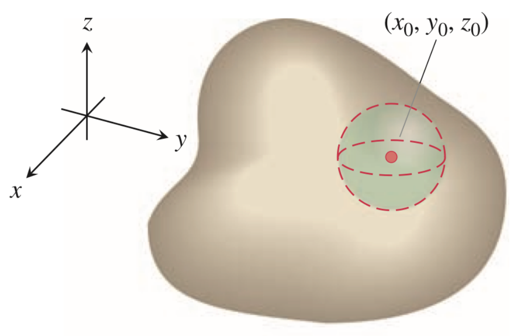

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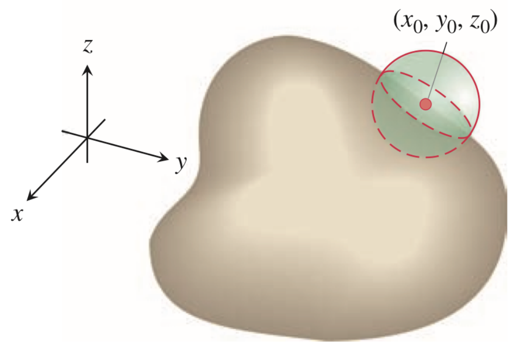

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- separable, metric space
- neighborhoods homeomorphic to $\mathbb{R}^n$ o $\mathbb{R}^n_+$
- ∂M=the boundary of M (the $\mathbb{R}^n_+$'s)
- $int(M)$=the interior of M (the $\mathbb{R}^n$'s)
- closed=connected, compact and no border

Some results for n=2, 3

Theorem (M. Dehn and P. Heegaard 1907; H.R. Brahana, 1921)

Every compact surface (closed 2-manifold) is homeomorphic to:

1. *a sphere* S^2, *or*
2. *a connected sum of tori* $T_1^2 \# T_2^2 \# \cdots \# T_k^2$, *or*
3. *a connected sum of projective planes* $P_1^2 \# P_2^2 \# \cdots \# P_k^2$.

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Theorem (H. Kneser 1929; J. Milnor, 1962)

Let M *be an orientable, closed 3-manifold. Then* M *has a decomposition*

$$M = M_1 \# M_2 \# \cdots \# M_k,$$

where each M_i *is prime. The collection* $\{M_i\}$ *is unique, except for the order of the factors.*

Some results for n=2, 3

Theorem (W. Jaco and P. Shalen 1979; K. Johannson, 1979)

Let M be an orientable, irreducible, closed 3-manifold. Then there is a finite and disjoint collection of incompressible tori $T_i^2 \subset M$ that separates M into a finite collection of compact 3-manifolds whose boundary consist of tori and each of which is a Seifert fibered space or atoroidal. Furthermore, the minimal such collection of tori is unique up to isotopies.

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Theorem (W. Thurston 1980)

There are exactly eight model geometries for 3-manifolds. Namely:

- S^3
- E^3
- H^3
- $S^2 \times E^1$
- $H^2 \times E^1$
- $\widetilde{SL(2,\mathbb{R})}$
- Nil
- Sol.

G. Perelman results 2002, 2003

Theorem (Hyperbolization)

Let M be an orientable, closed, prime 3-manifold. If $\pi_1(M)$ is infinite and M is atoroidal then M is hyperbolic.

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Theorem (Geometrization)

Let M be an orientable, closed, prime 3-manifold. Then there is a finite and disjoint collection of tori $\{T_i^2\}$ embedded in M, such that each component of $M - \{\bigcup T_i^2\}$ admits a geometric structure.

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Definition

Definition (Knot)

A knot K is an embbedding (p.l. or smooth) of $K : S^1 \longrightarrow S^3(\text{or } \mathbb{R}^3)$. More generally a knot K is an embbedding of $K : S^p \longrightarrow S^n$. It is common practice to write K for the image $K(S^1)$, or for the image $\pi \circ K(S^1)$, where $\pi : \mathbb{R}^3 \longrightarrow P$ is a projection onto some plane P.

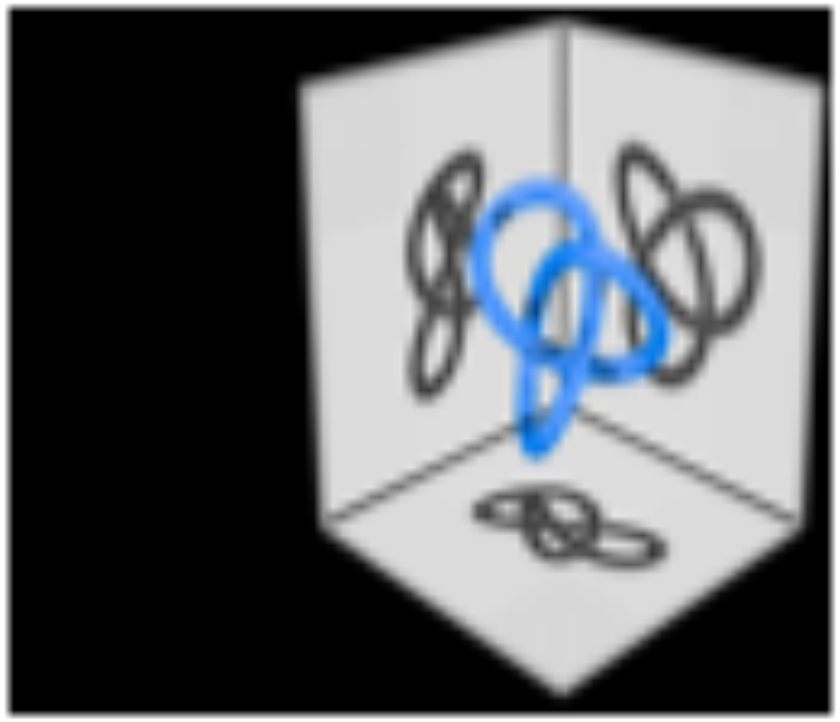

Figure: Trefoil projection.

Reidemeister and Δ-moves

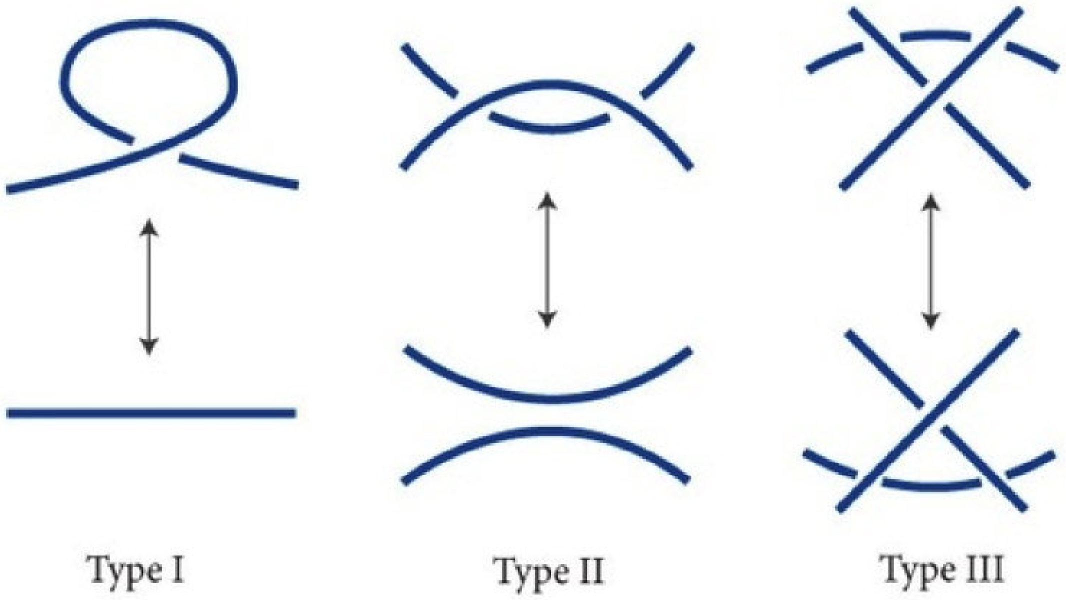

Figure: Reidemeister moves.

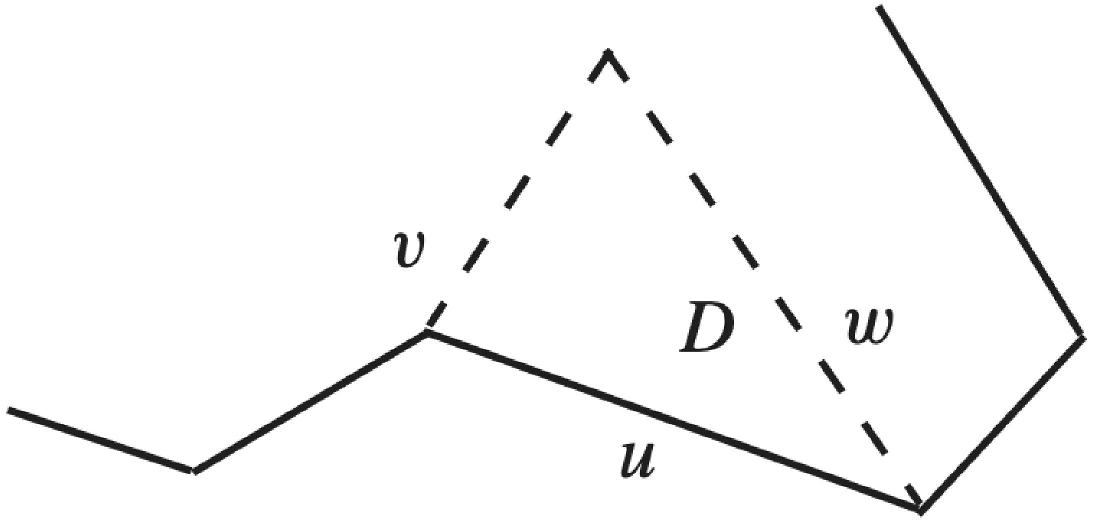

Figure: Δ-moves.

Equivalence of equivalences

Theorem

Let K_0, K_1 be p.l.-knots in S^3. The following are equivalent:

1. *There is an orientation preserving homeomorphism $h : S^3 \longrightarrow S^3$ which carries K_0 onto K_1, $h(K_0) = K_1$.*
2. *K_0 and K_1 are ambient isotopic.*
3. *K_0 and K_1 are isotopic by finitely many Δ-moves.*
4. *K_0 and K_1 are isotopic by finitely many Reidemeister moves.*

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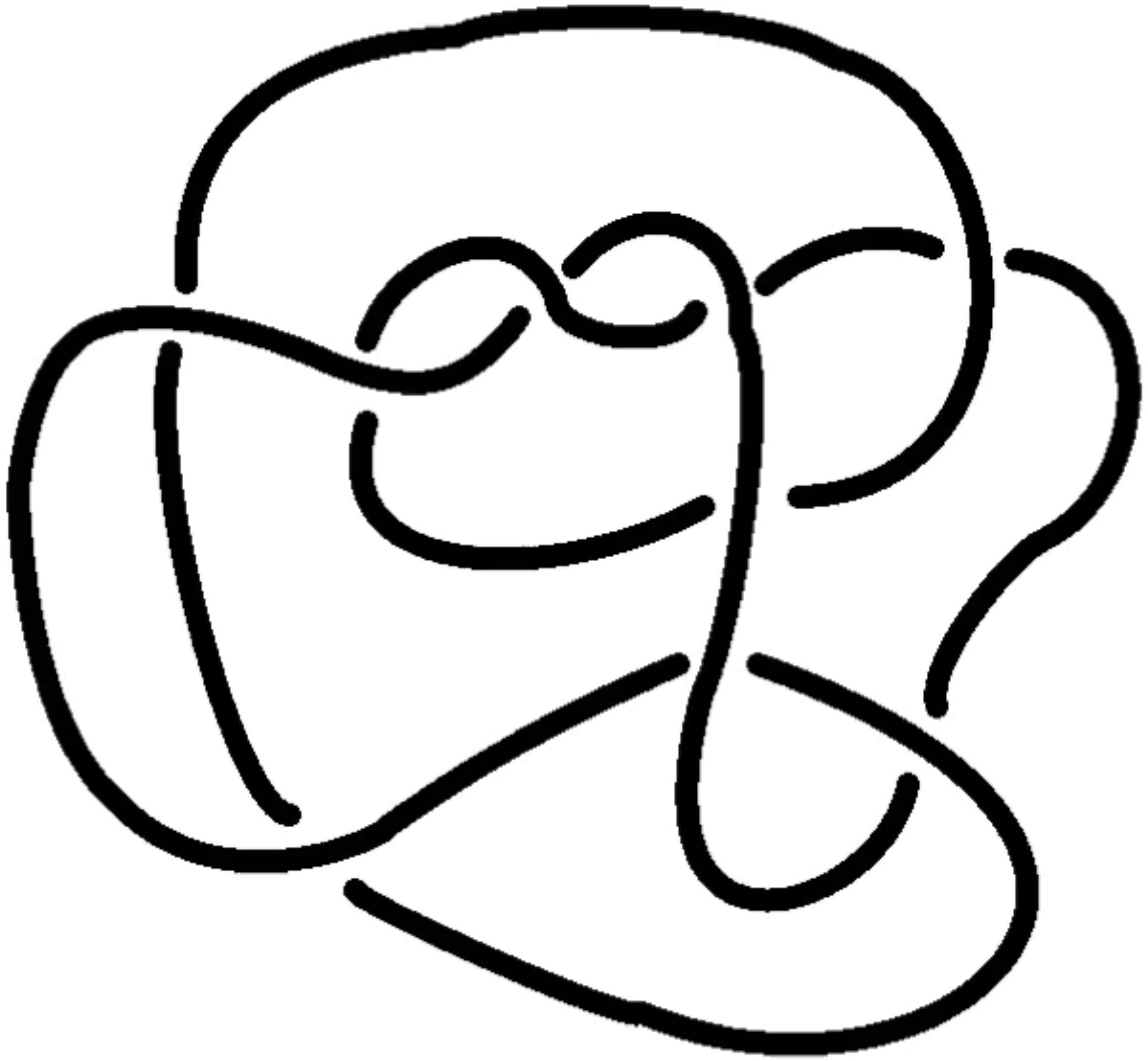
- *K_0 and K_1 are isotopic by finitely many Δ-moves.*
- *K_0 and K_1 are isotopic by finitely many Reidemeister moves.*

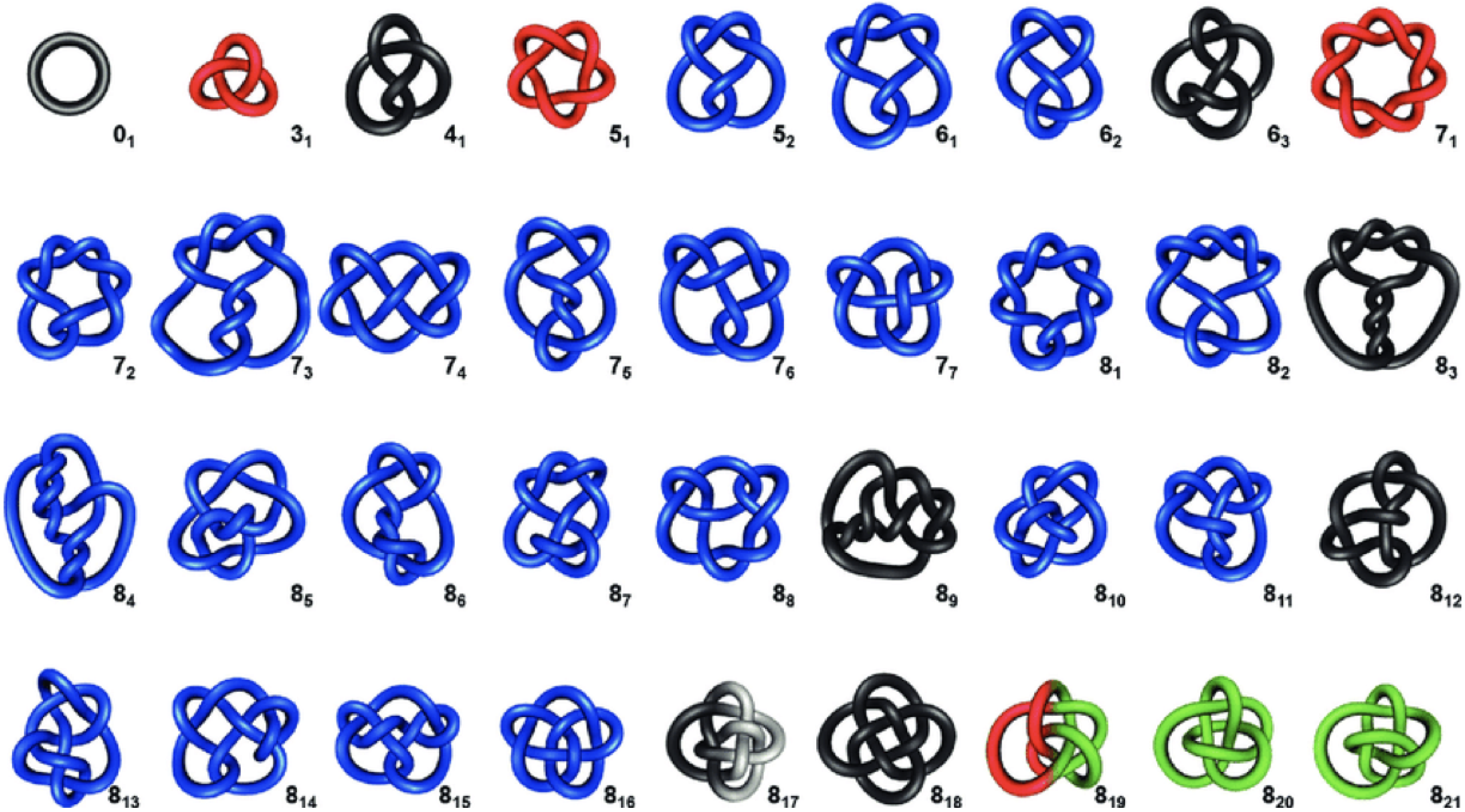
Theorem (Lickorish-Wallace 1960, 1962, 1963)

Let M be an orientable, closed, connected 3-manifold. Then M is obtained by surgery on some link K in S^3.

Can you detect the unknot?



Knots with less than 9 crossings



Pretzel knot $P(\pm l, \pm m, \pm n)$

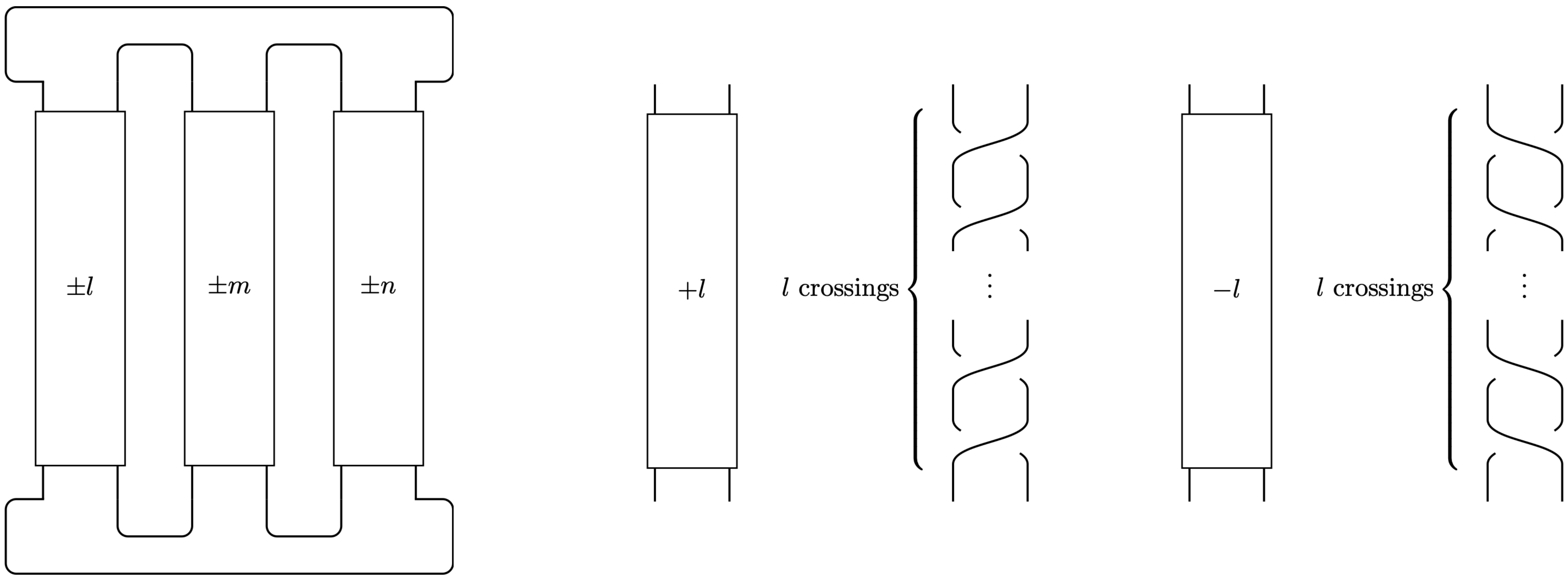


Figure: The pretzel knot $P(\pm l, \pm m, \pm n)$ with $l, m, n > 0$.

Pretzels I

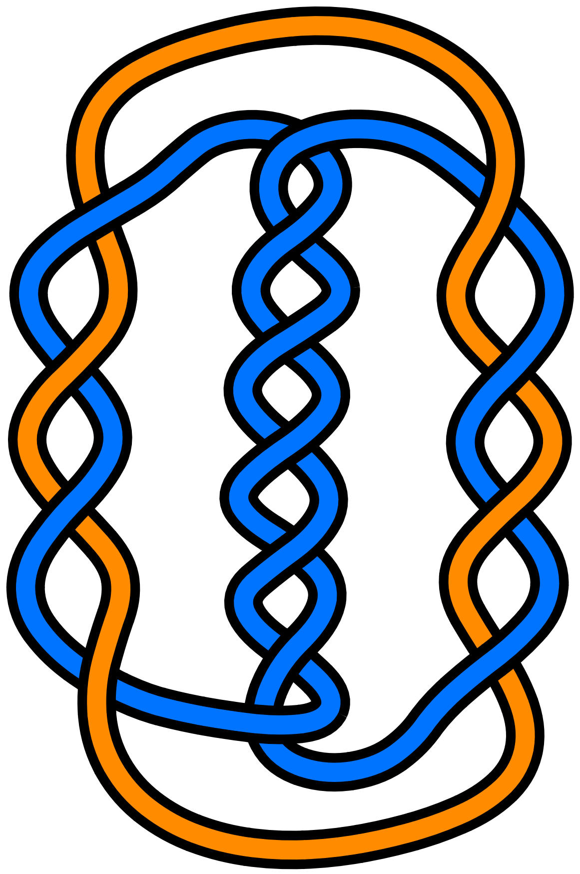


Figure: $P(4, 7, 4)$.

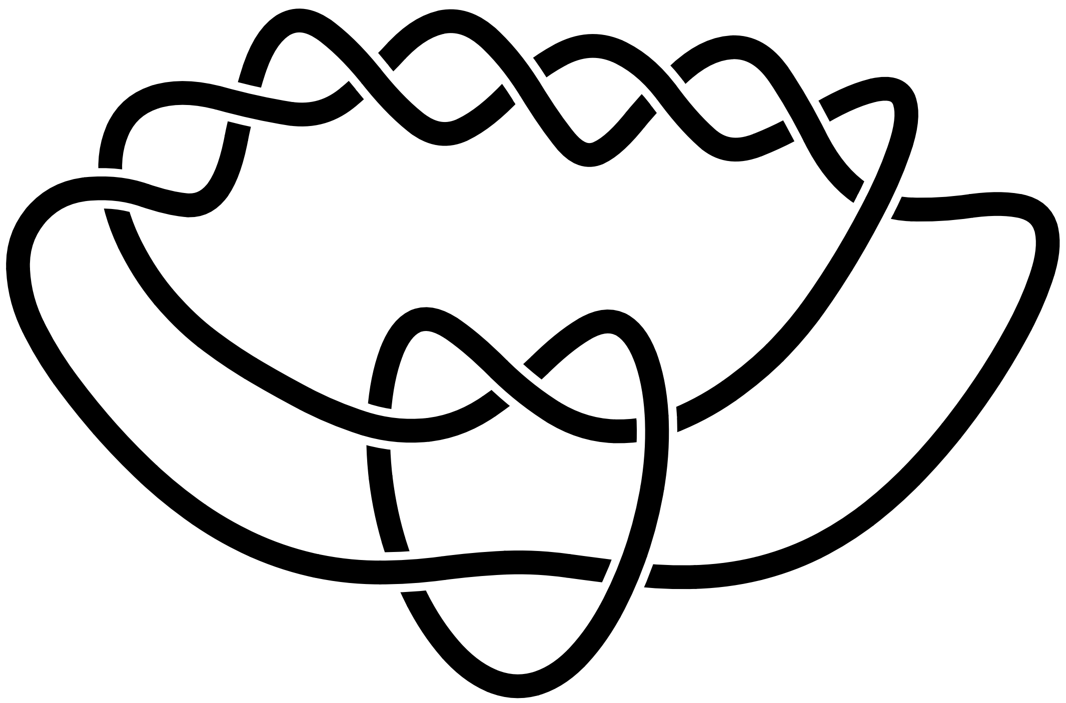


Figure: $P(-2, 3, 7)$.

Pretzels II

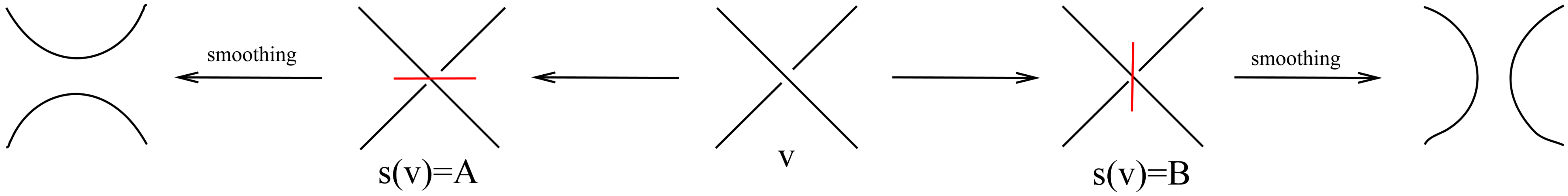


Figure: Markers at a crossing v of D and their corresponding smoothing.

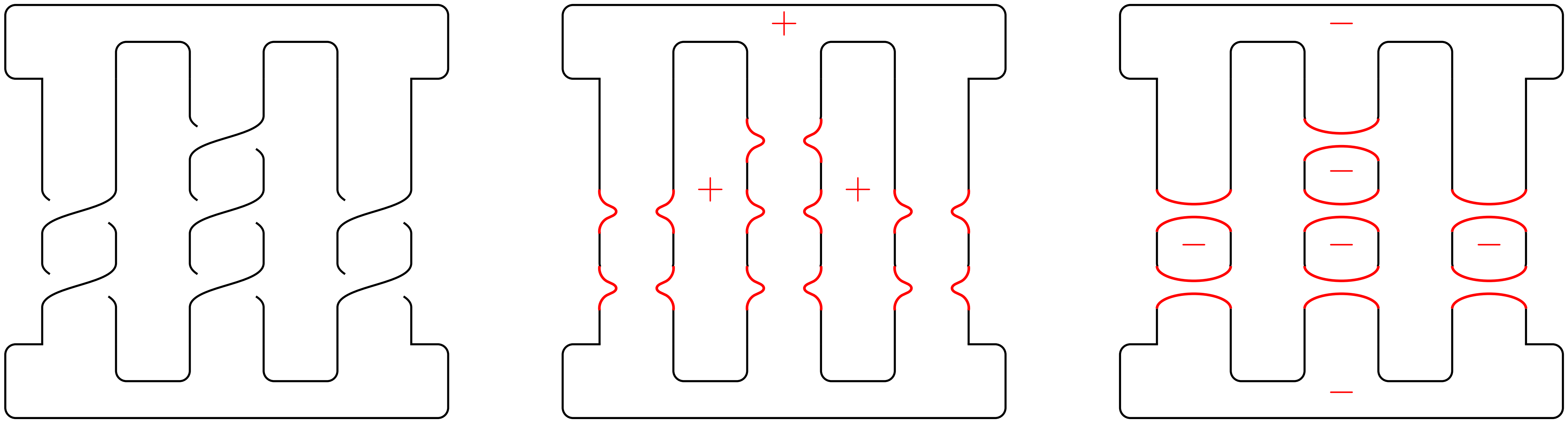


Figure: $P(2,3,2)$ and its S_A and S_B EKS's.

Pretzels III

The following knots up to 9 crossings are three column pretzels. See [Diaz, Manchón].

$$\begin{array}{lll} 3_1 = P(1,1,1) & 7_4 = P(3,1,3) & 9_2 = P(1,1,7) \\ 4_1 = P(1,1,2) & 8_1 = P(1,1,6) & 9_3 = P(1,-4,5) \\ 5_2 = P(1,1,3) & 8_2 = P(1,2,5) & 9_4 = P(1,-5,4) \\ 6_1 = P(1,1,4) & 8_4 = P(1,3,4) & 9_5 = P(1,3,5) \\ 6_2 = P(1,2,3) & 8_5 = P(2,3,3) & 9_{35} = P(3,3,3) \\ 7_2 = P(5,1,1) & 8_{19} = P(3,3,-2) & 9_{46} = P(3,3,-3) \end{array}$$

$8_{19} = P(3,3,-2) = T(4,3)$ and $10_{124} = P(5,3,-2) = T(5,3)$ are three column pretzel knots that are also torus knots.

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Free Abelian groups I

Given a set T, there is a free Abelian group F having T as its base. Any two bases for the free Abelian group F have the same cardinality.

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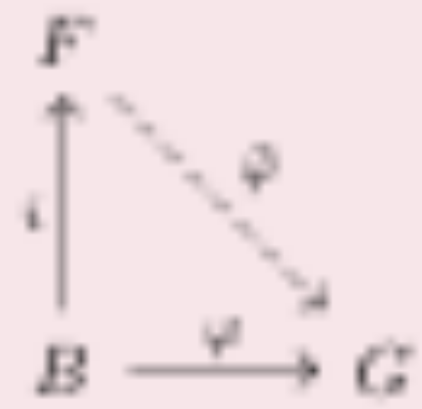
Theorem

Let F be a free Abelian group with base B.

1. *If G is an Abelian group and $\varphi : B \longrightarrow G$ is a function, then there is a unique homomorphism $\tilde{\varphi} : F \longrightarrow G$ with $\tilde{\varphi}(b) = \varphi(b)$ for all $b \in B$.*



2. *Every Abelian group G is isomorphic to a quotient group of the form F/R, where F is a free Abelian group.*

Free Abelian groups II

Examples

Let $T = \{@, \#, \%, \&\}$. The elements of $F[T]$, the free group with base T, have the form

$$\alpha = 2@ + 4\# + 3\% - \& \qquad \beta = 3@ - 4\# + \% \qquad \gamma = 8@ + 9\# + 10\% + 11\&.$$

Sumation is done formally, for example

$$\alpha + \beta + \gamma = 13@ + 9\# + 14\% + 10\&.$$

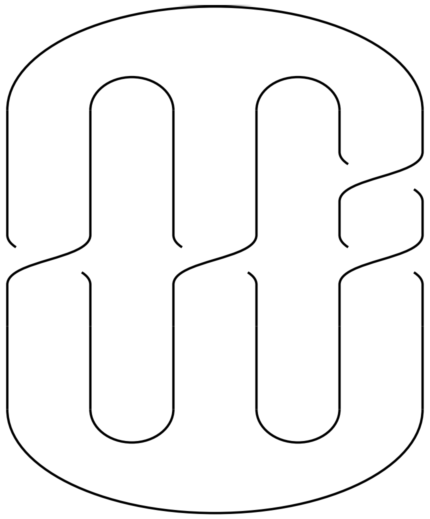
A typical element σ of $F[T]$ has the form

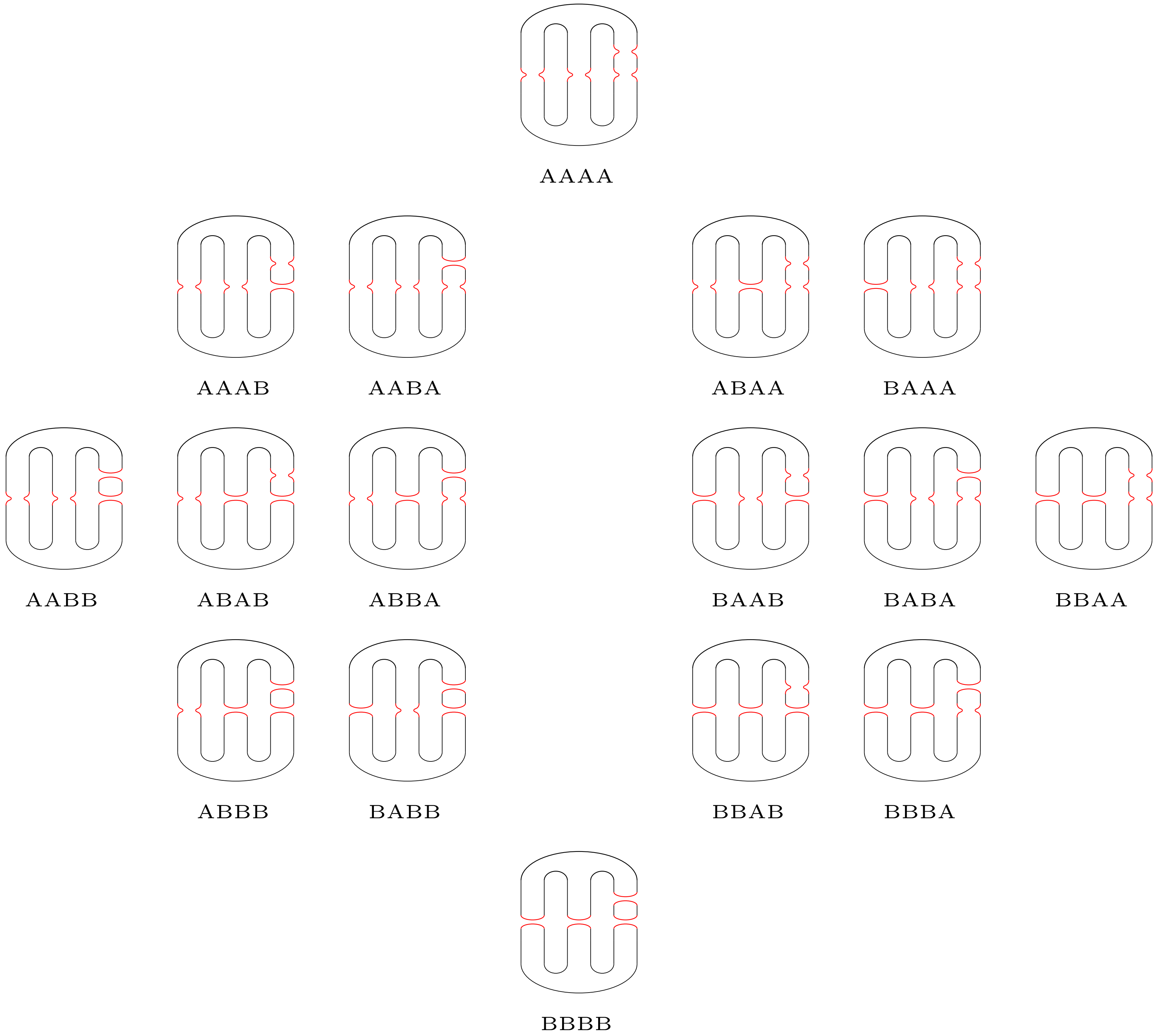
$$\sigma = n_1 @ + n_2 \# + n_3 \% + n_4 \&$$

where n_1, n_2, n_3, n_4 are integers. Here $F[T] \simeq \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$.

Free Abelian groups III

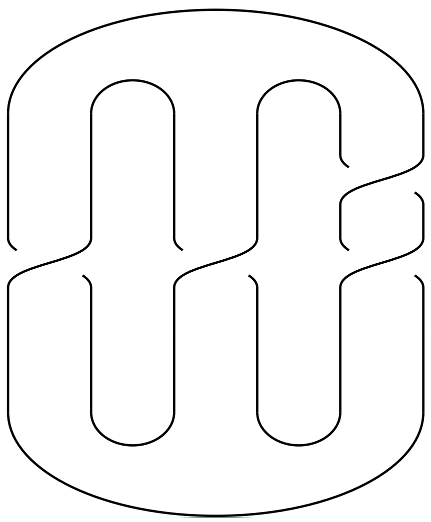
Kauffman states (KS) for $P(1,1,2) = 4_1$





Free Abelian groups IV

Enhanced Kauffman states (EKS) for $P(1,1,2) = 4_1$



AAAA: $\left\{\begin{array}{ccc} + & + & + \\ + & + & - \\ + & - & + \\ + & - & - \\ - & + & + \\ - & + & - \\ - & - & + \\ - & - & - \end{array}\right.$

AAAB: $\left\{\begin{array}{cc} + & + \\ + & - \\ - & + \\ - & - \end{array}\right.$ AABA: $\left\{\begin{array}{cc} + & + \\ + & - \\ - & + \\ - & - \end{array}\right.$ ABAA: $\left\{\begin{array}{cc} + & + \\ + & - \\ - & + \\ - & - \end{array}\right.$ BAAA: $\left\{\begin{array}{cc} + & + \\ + & - \\ - & + \\ - & - \end{array}\right.$

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O. Viro's (a, b) bigrading

For each EKS, we define the following:

$$\begin{aligned}
\sigma(s) &= |s^{-1}(A)| - |s^{-1}(B)| \\
\tau(S) &= |\varepsilon^{-1}(+)| - |\varepsilon^{-1}(-)| \\
a &= \sigma(s) \\
b &= \sigma(s) + 2\tau(S)
\end{aligned}$$

Table: Some generators of the chain groups with the (a, b) grading

S	σ	τ	a	b	generator
AAAA+++	4	3	4	10	$g^1_{(4,10)}$
AAAA+--	4	-1	4	2	$g^2_{(4,2)}$
ABBA+	0	1	0	2	$g^3_{(0,2)}$
BAAB-	0	-1	0	-2	$g^4_{(0,-2)}$
BBAA-	0	-1	0	-2	$g^5_{(0,-2)}$
BBBB---	-4	-3	-4	-10	$g^6_{(-4,-10)}$

Khovanov Homology chain complex I

Definition

Let D be an unoriented link diagram and let $cr(D)$ be its crossings set. A **Kauffman state** *s, of D, is a function $s : cr(D) \longrightarrow \{A, B\}$. This function is understood as an assignment of a* **marker** *to each crossing according to the following illustration:*

Figure: Markers at a crossing v of D and their corresponding smoothing.

Khovanov Homology chain complex II

Definition

An **enhanced Kauffman state** S *of* D *is a Kauffman state* s *together with a function* $\varepsilon : D_s \longrightarrow \{+, -\}$, *assigning to each circle of* D_s *a positive or a negative sign.*

Khovanov Homology chain complex II

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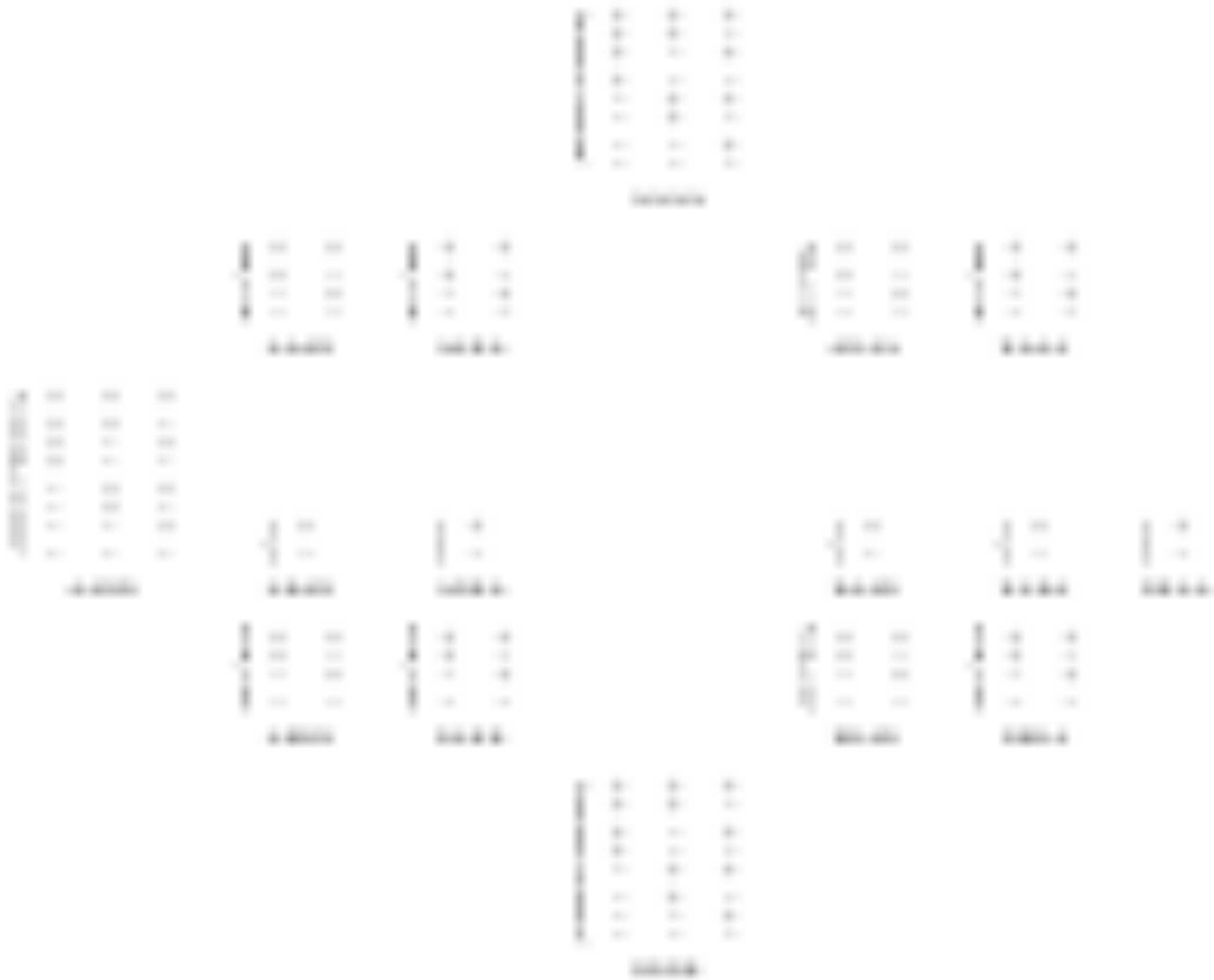

Khovanov Homology chain complex III

Definition

(i) *The* **bidegree** *on the enhanced Kauffman states is defined as the following set:*

$$S_{a,b}(D) = S_{a,b} = \{S \in EKS \mid a = \sigma(s),\ b = \sigma(s) + 2\tau(S)\}.$$

(ii) *The* **chain groups** $\mathcal{C}_{a,b}(D) = \mathcal{C}_{a,b}$, *are defined to be the free abelian groups with basis* $S_{a,b}(D) = S_{a,b}$, *i.e.* $\mathcal{C}_{a,b} = \mathbb{Z}S_{a,b}$. *Therefore,* $\mathcal{C}(D) = \bigoplus_{a,b \in \mathbb{Z}} \mathcal{C}_{a,b}(D)$ *is a bigraded free abelian group.*

(iii) *For a link diagram D we define the* **chain complex** $C(D) = \{(C_{a,b}, \partial_{a,b})\}$, *where the* **differential map** $\partial_{a,b} : \mathcal{C}_{a,b} \longrightarrow \mathcal{C}_{a-2,b}$ *is defined by*

$$\partial_{a,b}(S) = \sum_{S' \in S_{a-2,b}} (-1)^{t(S,S')}(S, S')S'.$$

Khovanov Homology chain complex IV

Definition

The **Khovanov homology** *of the diagram D is defined to be the homology of the chain complex $C(D)$:*

$$H_{a,b}(D) = \frac{ker(\partial_{a,b})}{im(\partial_{a+2,b})}.$$

Khovanov Homology chain complex IV

Definition

The **Khovanov homology** *of the diagram D is defined to be the homology of the chain complex $C(D)$:*

$$H_{a,b}(D) = \frac{ker(\partial_{a,b})}{im(\partial_{a+2,b})}.$$

Theorem

Let D be an unoriented link diagram. The homology groups $H_{a,b}(D)$ are invariant under Reidemeister moves of second and third type. Therefore, they are invariants of unoriented framed links. Moreover, the effect of the first Reidemeister move R_1, is the shift in the homology. $H_{a,b}(R_{1+}(D)) = H_{a+1,b+3}(D)$ and $H_{a,b}(R_{1-}(D)) = H_{a-1,b-3}(D)$. These groups categorify the unreduced Kauffman bracket polynomial and are called the framed Khovanov homology groups.

Exact sequences I

Theorem (O. Viro 2002)

The following sequence is exact:

$$\cdots \longrightarrow H_{a+1,b+1}(\times) \xrightarrow{\alpha_*} H_{a,b}(\times) \xrightarrow{\beta_*} H_{a-1,b-1}(\times)$$
$$\xrightarrow{(\partial^{Conn}_{a,b})_*} H_{a-1,b+1}(\times) \xrightarrow{\alpha_*} H_{a-2,b}(\times) \xrightarrow{\beta_*} H_{a-3,b-1}(\times) \quad (1)$$
$$\xrightarrow{(\partial^{Conn}_{a,b})_*} H_{a-3,b+1}(\times) \xrightarrow{\alpha_*} H_{a-4,b}(\times) \xrightarrow{\beta_*} \cdots .$$

Exact sequences II

Theorem (M. M. Asaeda, J. H. Przytycki 2004)

The following sequence is exact:

$$0 \to H_{i+2,j+4}(D) \xrightarrow{\alpha_h} H_{i,j}(D\#D_h) \xrightarrow{\beta_h} H_{i-2,j-4}(D) \to 0$$

.

Theorem (M. M. Asaeda, J. H. Przytycki 2004)

The above short exact sequence of homology splits, so we have

$$H_{i,j}(D\#D_h) = H_{i+2,j+4}(D) \oplus H_{i-2,j-4}(D).$$

Diagonals property

Theorem (E.S. Lee 2002)

For any alternating knot L, the Khovanov invariants $\mathcal{H}^{i,j}(L)$ of L are supported in two lines

$$j = 2i - \sigma(L) \pm 1 \ ,$$

where $\sigma(L)$ is the signature of L.

Since the classical Khovanov cohomology $\mathcal{H}^{i,j}(\vec{L})$, and the framed version of KH $H_{a,b}(L)$, are related by the following equalities:

$$\mathcal{H}^{i,j}(\vec{L}) = H_{w-2i,3w-2j}(L) = H_{a,b}(L) = \mathcal{H}^{\frac{w-a}{2},\frac{3w-b}{2}}(\vec{L}),$$

where $w(\vec{L}) = w$ is the writhe of the oriented link diagram $\vec{L}$, in terms of the framed version of KH and the (a, b) grading, these supporting lines are

$$b = 2a + w(\vec{L}) + 2\sigma(L) \pm 2.$$

KH of the knot $4_1 = P(1,1,2)$

P(1,1,2)

	$a=-4$	$a=-2$	$a=0$	$a=2$	$a=4$
$b=10$					$\mathbb{Z}$
$b=6$				$\mathbb{Z}_2$	0
$b=2$			$\mathbb{Z}$	$\mathbb{Z}$	
$b=-2$		$\mathbb{Z}$	$\mathbb{Z}$		
$b=-6$	$\mathbb{Z}_2$	0			
$b=-10$	$\mathbb{Z}$				

KH of the knot $P(1,1,7)$

P(1,1,7)		$i=0$ $r=0$ $a=-9$	$i=-1$ $r=-1$ $a=-7$	$i=-2$ $r=-2$ $a=-5$	$i=-3$ $r=-3$ $a=-3$	$i=-4$ $r=-4$ $a=-1$	$i=-5$ $r=-5$ $a=1$	$i=-6$ $r=-6$ $a=3$	$i=-7$ $r=-7$ $a=5$	$i=-8$ $r=-8$ $a=7$	$i=-9$ $r=-9$ $a=9$
$j=-21$	$b=15$										$\mathbb{Z}$
$j=-19$	$b=11$									$\mathbb{Z}_2$	
$j=-17$	$b=7$								$\mathbb{Z}$	$\mathbb{Z}$	
$j=-15$	$b=3$							$\mathbb{Z}\oplus\mathbb{Z}_2$			
$j=-13$	$b=-1$						$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}$			
$j=-11$	$b=-5$					$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}$				
$j=-9$	$b=-9$				$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}$					
$j=-7$	$b=-13$			$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}$						
$j=-5$	$b=-17$		$\mathbb{Z}_2$	$\mathbb{Z}$							
$j=-3$	$b=-21$	$\mathbb{Z}$	$\mathbb{Z}$								
$j=-1$	$b=-25$	$\mathbb{Z}$									

Diagonal lines labelled: $i=-5$, $i=-3$, $i=-1$, $i=1$

KH of the knot $P(1,1,7)$

P(1,1,7)		$i=0$, $r=0$, $a=-9$	$i=-1$, $r=-1$, $a=-7$	$i=-2$, $r=-2$, $a=-5$	$i=-3$, $r=-3$, $a=-3$	$i=-4$, $r=-4$, $a=-1$	$i=-5$, $r=-5$, $a=1$	$i=-6$, $r=-6$, $a=3$	$i=-7$, $r=-7$, $a=5$	$i=-8$, $r=-8$, $a=7$	$i=-9$, $r=-9$, $a=9$
$j=-21$	$b=15$										$\mathbb{Z}$
$j=-19$	$b=11$									$\mathbb{Z}_2$	
$j=-17$	$b=7$								$\mathbb{Z}$	$\mathbb{Z}$	
$j=-15$	$b=3$							$\mathbb{Z}\oplus\mathbb{Z}_2$			
$j=-13$	$b=-1$						$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}$			
$j=-11$	$b=-5$					$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}$				
$j=-9$	$b=-9$				$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}$					
$j=-7$	$b=-13$			$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}$						
$j=-5$	$b=-17$		$\mathbb{Z}_2$	$\mathbb{Z}$							
$j=-3$	$b=-21$	$\mathbb{Z}$	$\mathbb{Z}$								
$j=-1$	$b=-25$	$\mathbb{Z}$									

$\Longrightarrow$ We use $P116$ & $(\mathrm{O}^{-6}\#D_h)$ & LES

Manually & T28 & T27 $\Longleftarrow$

$i=-5$

$i=-3$

$i=-1$

$i=1$

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- the unknot (P. B. Kronheimer and T. S. Mrowka, 2010)

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- the figure 8 knot $4_1 = P(1,1,2)$ (J. A. Baldwin, N. Dowlin, A. S. Levine, T. Lidman, R. Sazdanovic, 2020)

Thanks for you time.